WEEKLY TEST OYJ TEST - 15
SOLUTION Date 21-07-2019

[PHYSICS]

1. The magnetic moment will be zero because it becomes an open circuit.

2. $W = MB(1 - \cos\theta)$

and $= MB(1 - \cos 180°) = 2MB$

3. The angle which the total magnetic field of earth makes with the surface of the earth is called Geographic Meridian.

4. Magnetic moment

$$M = \frac{\tau}{B\sin\theta} = \frac{0.032}{0.16 \times \sin 30°} = 0.40\ JT^{-1}$$

5. Inside bar magnet, lines of force are from south to north.

6. Intensity of magnetisation

$$I = \frac{M}{V} = \frac{M}{\text{mass / density}}$$

Given mass $= 1\ g = 10^{-3}$ kg

and density $= 5 g / cm^3 = \frac{5 \times 10^{-3} kg}{(10^{-2})^3}$

$$= 5 \times 10^3 kg / m^3$$

Hence, $I = \frac{6 \times 10^{-7} \times 5 \times 10^3}{10^{-3}}$

$$= 3\ A/m$$

7. (a) Let the real dip be ϕ, then $\tan\phi = \frac{B_V}{B_H}$

For apparent dip,

$$\tan\phi' = \frac{B_V}{B_H \cos\beta} = \frac{B_V}{B_H \cos 30^o} = \frac{2B_V}{\sqrt{3}B_H}$$

or $\tan 45^o = \frac{2}{\sqrt{3}} . \tan\phi$ or $\phi = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

8. (b) $\tan\phi' = \frac{\tan\phi}{\cos\beta}$; where ϕ' = Apparent angle of dip,

ϕ = True angle of dip, β = Angle made by vertical plane with magnetic meridian.

$$\Rightarrow \tan\phi' = \frac{\tan 60^o}{\cos 30^o} = 2 \Rightarrow \phi' = \tan^{-1}(2)$$

9. (c) Initially magnetic moment of system

$M_1 = \sqrt{M^2 + M^2} = 2M$ and moment of inertia

$I_1 = I + I = 2I.$

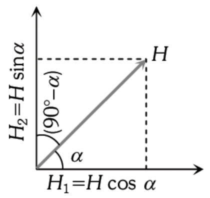
Finally when one of the magnet is removed then

$M_2 = M$ and $I_2 = I$

So $T = 2\pi\sqrt{\frac{I}{M\,B_H}}$

$$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2} \times \frac{M_2}{M_1}} = \sqrt{\frac{2I}{I} \times \frac{M}{\sqrt{2}M}} \Rightarrow T_2 = \frac{2^{5/4}}{2^{1/4}} = 2 \text{ sec}.$$

10. (d) Let α be the angle which one of the planes make with the magnetic meridian the other plane makes an angle $(90^o - \alpha)$ with it. The components of H in these planes will be $H\cos\alpha$ and $H\sin\alpha$ respectively. If ϕ_1 and ϕ_2 are the apparent dips in these two planes, then



$$\tan\phi_1 = \frac{V}{H\cos\alpha} \quad i.e. \quad \cos\alpha = \frac{V}{H\tan\phi_1} \quad \text{..... (i)}$$

$$\tan\phi_2 = \frac{V}{H\sin\alpha} \quad i.e. \quad \sin\alpha = \frac{V}{H\tan\phi_2} \quad \text{..... (ii)}$$

Squaring and adding (i) and (ii), we get

$$\cos^2\alpha + \sin^2\alpha = \left(\frac{V}{H}\right)^2\left(\frac{1}{\tan^2\phi_1} + \frac{1}{\tan^2\phi_2}\right)$$

$$i.e. \quad 1 = \frac{V^2}{H^2}\left(\cot^2\phi_1 + \cot^2\phi_2\right)$$

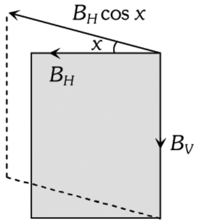
$$\text{or} \quad \frac{H^2}{V^2} = \cot^2\phi_1 + \cot^2\phi_2 \quad i.e. \quad \cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

This is the required result.

11. (a) In first case $\tan\theta = \frac{B_V}{B_H}$ ..... (i)

Second case $\tan\theta' = \frac{B_V}{B_H \cos x}$ ..... (ii)

From equation (i) and (ii), $\frac{\tan\theta'}{\tan\theta} = \frac{1}{\cos x}$



12. We have

$$r = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{2mV}{aB^2}}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}$$

$$\frac{m_1}{m_2} = \frac{r_1^2}{r_2^2}$$

Hence, $\frac{m_1}{m_2} = \frac{(2)^2}{(3)^2} = \frac{4}{9}$

13.

14. Given $\theta = 23°$, $B = 2.6 \text{ mT} = 2.6 \times 10^{-6}$ T

and $F = 6.5 \times 10^{-17}$ N

We know

$$F = qvB\sin\theta$$

$$6.5 \times 10^{-17} = 1.6 \times 10^{-19} \times v \times 2.6 \times 10^{-6} \times \sin 23°$$

$$v = \frac{6.5 \times 10^{-17}}{2.6 \times 10^{-6} \times 1.6 \times 10^{-19} \times 0.39}$$

$$v = 4 \times 10^5 \text{ ms}^{-1}$$

15. Radius, $r = \frac{mv}{qB}$

or $B = \frac{mv}{qr}$

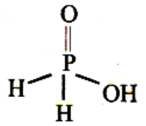
$$= \frac{9.1 \times 10^{-31} \times 1.3 \times 10^6}{1.6 \times 10^{-19} \times 0.35}$$

$$= 2.1 \times 10^{-5} \text{ T}$$

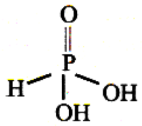
[CHEMISTRY]

16.

(b) Phosphinic acid as shown in structure below has one P—OH bend thus it is monobasic or monoprotic


(Monoprotic)

Phosphonic acid as shown in structure has two P–OH bonds thus it is dibasic or diprotic

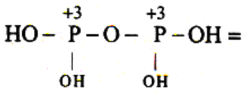

(Diprotic acid)

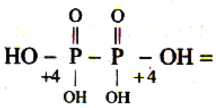
17.

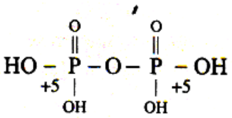
(a) The acids which contain P-H bond have strong reducing properties. Thus H_3PO_2 acid is good reducing agent as it contains two P–H bonds and reduces, for example, $AgNO_3$ to metallic silver.

$$4\,AgNO_3 + 2H_2O + H_3PO_2 \longrightarrow 4Ag + 4HNO_3 + H_3PO_4$$

18

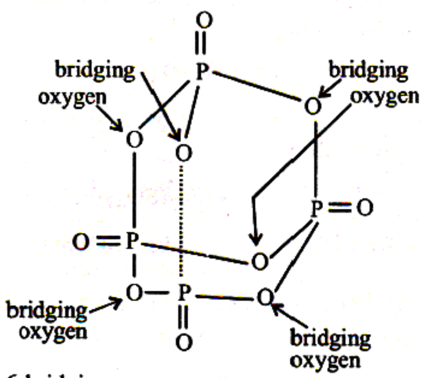
(a)  $= H_4P_2O_5$

 $= H_4P_2O_6$

 $= H_4P_2O_7$

19.

(b)



i.e. 6-bridging oxygen.

20.

(c) Nitrogen form N_2 (i.e. $N \equiv N$) but phosphorus form P_4, because in P_2, $p_\pi - p_\pi$ bonding is present which is a weaker bonding.

21.

(a) Order of dipole moment decreases as
$NH_3 > PH_3 > AsH_3 > SbH_3$
(Based upon electronegativity)

22.

(b) If acidic nature is high, K_a is high and pK_a is low

	H_2O	H_2S
K_a	1.8×10^{-6}	1.3×10^{-7}
	H_2Se	H_2Te
K_a	1.3×10^{-4}	2.3×10^{-3}

since $pK_a = -\log K_a$
Hence the order of pK_a will be
$H_2O > H_2S > H_2Se > H_2Te$

23.

(d) We know that positive ion is always smaller and negative ion is always larger than the corresponding atom. Therefore the correct order of the size is $I^- > I > I^+$

24.

(d) The halide ions act as reducing agents . F^- ion does not show any reducing nature but Cl^-, Br^- & I^- ion act as reducing agents and their reducing nature is in increasing order

$$\xrightarrow[\text{Reducing nature increases}]{Cl^- \quad Br^- \quad I^-}$$

25.

(b) Bond dissociation enthalpy decreases as

the bond distance increases from F_2 to I_2. This is due to increase in the size of the atom, on moving from F to I.
F – F bond dissociation enthalpy is smaller then Cl – Cl and even smaller than Br – Br. This is because F atom is very small and hence the three lone pairs of electrons on each F atom repel the bond pair holding the F-atoms in F_2 molecules.
The increasing order of bond dissociation enthalphy is
$I_2 < F_2 < Br_2 < Cl_2$

26.

(b) $S + O_2 \longrightarrow SO_2$ (burns with blue light)

$4Na + O_2 \longrightarrow 2NaO$

(burns with yellow light)

$P_4 + 3O_2 \longrightarrow P_4O_6$

$P_4 + 5O_2 \longrightarrow P_4O_{10}$

$Cl + O_2 \longrightarrow$ No reaction

Chlorine does not react directly with oxygen.

27.

(a) The weaking of M—H bond with increase in size of M (where M = S, Se, Te) explains the acid character of hydrides. Since on moving down the group atomic size increases hence bond length increases and hence removal tendency of H also increases.

28.

(a) As the O.N of the central atom of the compounds increases acidic strength of that compound also increases and on moving from top to bottom in groups acidic strength of oxides also decrease due to decreasing electronegativity in groups.

$$\overset{+5}{P_4O_{10}} > \overset{+3}{P_4O_6} > \overset{+5}{As_4O_{10}} > \overset{+3}{As_4O_6}$$

29.

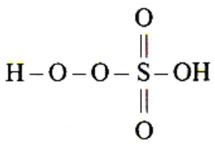
(a) $2KMnO_4 + 5H_2S + 3H_2SO_4 \longrightarrow$

$K_2SO_4 + 2MnSO_4 + 5S + 8H_2O.$

Thus in this reaction sulphur (S) is produced.

89.

(d) Caro's acid is H_2SO_5 which contains one $S-O-O-H$ peroxy linkage. It is also known as permonosulphuric acids.



Caro's acid

30.

(c) H_3PO_2 is named as hypophosphorous acid. As it contains only one $P-OH$ group, its basicity is one.

[MATHEMATICS]

31.

$y = x + 1$ will be || to tangent at $P(y_0^2, y_0)$ on $x = y^2$

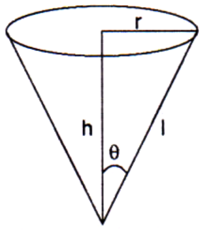
$\Rightarrow \frac{1}{2y_0} = 1 \Rightarrow y_0 = \frac{1}{2}$

$\Rightarrow P\left(\frac{1}{4}, \frac{1}{2}\right)$

$\Rightarrow$ ⊥r distance of line from P $\equiv \frac{\left|\frac{1}{4} - \frac{1}{2} + 1\right|}{\sqrt{(1)^2 + (-1)^2}} = \frac{3/4}{\sqrt{2}} =$
$\frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

32.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\left(\sqrt{\ell^2 - r^2}\right)$



$\therefore \frac{dV}{dr} = \frac{\pi}{3}\left[r^2\left(\frac{-2r}{2\sqrt{\ell^2 - r^2}}\right) + 2r\sqrt{\ell^2 - r^2}\right]$

$= \frac{\pi}{3}\left[\frac{-r^3 + 2r\left(\ell^2 - r^2\right)}{\sqrt{\ell^2 - r^2}}\right] = \frac{\pi}{3}\left[\frac{2r\ell^2 - 3r^3}{\sqrt{\ell^2 - r^2}}\right]$

$\Rightarrow$ For maximum volume $\frac{dV}{dr} = 0$

$\Rightarrow r(2\ell^2 - 3r^2) = 0$

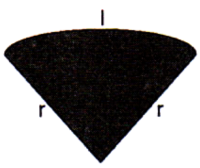
$\Rightarrow \frac{\ell^2}{r^2} = \frac{3}{2} \Rightarrow \frac{\ell}{r} = \sqrt{\frac{3}{2}}$

$\Rightarrow \operatorname{cosec}\theta = \sqrt{3/2}$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{2}} \Rightarrow \tan\theta = \sqrt{2}$

33.

$P = 60^0 = \ell + 2r$ ...(1)



Also $A = \frac{1}{2} = \ell r = \frac{1}{2}(60 - 2r)r$

$\Rightarrow A = 30r - r^2$

$\Rightarrow \frac{dA}{dr} = 30 - 2r$ and $\frac{d^2A}{dr^2} = -2$

$\therefore \quad \frac{dA}{dr} = 0$

$\Rightarrow \ell = 15$

$\therefore$ Maximum Area for $\ell = 15$

34.

$x + y = 12$ .....(1)

$P = x^2 .(y)^4$

$\Rightarrow P = y^4(12 - y)^2$

$\Rightarrow \frac{dP}{dy} = y^4(2)(12 - y)(-1) + (12 - y)^2 . 4y^3$

$\therefore \frac{dP}{dy} = 0$

$\Rightarrow 2(12 - y). y^3 (24 - 2y - y) = 0$

$\Rightarrow 2(12 - y) . y^3 (24 - 3y) = 0$

$\Rightarrow y = 0$ or 12 or 8

$\Rightarrow y$ must be 8

$\Rightarrow$ Two parts will be 4 and 8

35.

$S = 2x + 3y;\ xy = 6$

$\Rightarrow S = 2x + 3\left(\frac{6}{x}\right) = 2x + \frac{18}{x}$

$\therefore \frac{dS}{dx} = 2 + 18\left(\frac{-1}{x^2}\right)$

$\therefore \frac{dS}{dx} = 0 \Rightarrow 1 = \frac{9}{x^2}$

$\Rightarrow x = 3$ or -3

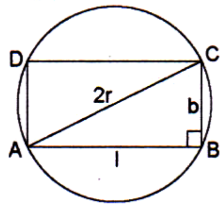
$\frac{d^2S}{dx^2} = \frac{36}{x^3}$ which > 0 for $x = 3$

$\Rightarrow x = 3, y = 2$

$\Rightarrow S_{min} = 2(3) + 3(2) = 12$

36.

Area of rectangle, $A = \ell b$ or $A = \ell.\sqrt{4r^2 - \ell^2}$



$\Rightarrow \frac{dA}{d\ell} = \frac{\ell(-2\ell)}{2\sqrt{4r^2 - \ell^2}} + \sqrt{4r^2 - \ell^2}$

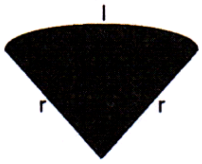
$= \frac{-\ell^2 + 4r^2 - \ell^2}{\sqrt{4r^2 - \ell^2}} = \frac{4r^2 - 2\ell^2}{\sqrt{4r^2 - \ell^2}}$

$\therefore \frac{dA}{d\ell} = 0 \quad \Rightarrow 4r^2 = 2\ell^2$

$\Rightarrow \ell^2 = 2r^2 \quad \therefore A = \sqrt{2}r\sqrt{2r^2} = 2r^2$

37.

$P = 2r + \ell = 2r + r\theta = (2 + \theta)r$



$$A = \frac{\theta . r^2}{2} = \frac{\theta}{2}\left(\frac{p}{2+\theta}\right)^2 ; p = \text{constant}$$

$$\Rightarrow \frac{dA}{d\theta} = \frac{1}{2}p^2\left(\frac{\theta}{(2+\theta)^2}\right) = \frac{1}{2}p^2\left[\frac{(2+\theta)^2 - \theta . 2(2+\theta)}{(2+\theta)^4}\right]$$

$$= \frac{1}{2}p^2\left[\frac{(2+\theta)(2+\theta-2\theta)}{(2+\theta)^4}\right] = \frac{1}{2}p^2 = \frac{(2-\theta)}{(2+\theta)^3} = 0$$

$\Rightarrow \theta = 2^c$

38.

$f(x) = x^3 + bx^2 + ax + 5; x \in [1, 3]$

$c = \left(2 + \frac{1}{\sqrt{3}}\right)$, $(a, b) =$? and $f(1) = f(3)$

$\Rightarrow 1 + b + a + 5 = 27 + 9b + 3a + 5$

$\Rightarrow a + b + 1 = 3a + 9b + 27$

$\Rightarrow 2a + 8b = -26$

$\Rightarrow a + 4b = -13$ ...(1)

Now $f'(x) = 3x^2 + 2bx + a$

$f'(c) = 0$

$\Rightarrow 3c^2 + 2bc + a = 0$ ...(2)

$\because c = 2 + \frac{1}{\sqrt{3}}$ is a root of equation (2)

$\Rightarrow c = 2 - \frac{1}{\sqrt{3}}$ is also a root of equation (2)

$\Rightarrow \frac{-2b}{3} = 4; \; 4 - \frac{1}{3} = \frac{a}{3}$

$\Rightarrow a = 11, b = -6$

$\Rightarrow (a, b) = (11, -6)$

39.

(a) $f(x) = \tan x$ is discontinuous in $[0, \pi]$, having discontinuity at $\pi/2$

$\Rightarrow f(x) = \tan x$ can't be so.

(b) Next, $f(x) = \cos(1/x); x \in [-1, 1]$

$f(x)$ has discontinuity (oscillating at $x = 0$,

(c) $f(x) = x^2$ in $[2, 3]$

$f(2) = 4 \neq f(3) = 9$

(d) $f(x) = x(x + 3) e^{-x/2}$ in $[-3, 0]$

$f(-3) = f(0) = 0$,

$f(x)$ is continuous $\forall x \in [-3, 0]$,

Also $f(x)$ is differentiable $\forall x \in (-3, 0)$.

Hence Rolle's theorem is applicable

40.
Let $f(x) = ax^3 + bx^2 + cx$

$\Rightarrow f(0) = f(1) = a + b + c = 0$

$\therefore$ By Rolle's Theorem, $f'(\alpha) = 0$ for at least one $\alpha \in (0, 1)$

$\Rightarrow 3ax^2 + 2bx + c = 0$ has at least one root in $(0, 1)$.

41.

$f(x) = \ell n\, x;\ x \in [1, 3]$

By L.M.V.T, $f'(c) = \dfrac{f(3) - f(1)}{3 - 1}$ for some $c \in (1, 3)$

$\Rightarrow 2f'(c) = \ell n\, 3 - \ell n\, 1 = \ell n\, 3$

$\Rightarrow f'(c) = \dfrac{1}{2} \ell og_e 3$

42.

By L.M.V.T, $\exists\ c \in (0, 2)$ such that $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$

i.e., $f'(c) = \dfrac{f(2)}{2}$

$\because\ |f'(x)| \le 1/2\ \forall\ x \in [0, 2]$

$\Rightarrow |f'(c)| \le \dfrac{1}{2}$

$\Rightarrow \left|\dfrac{f(2)}{2}\right| \le \dfrac{1}{2}$

$\Rightarrow |f(2)| \le 1$

Similarly applying L.M.V.T on $[0, x]$; where $x \in (0, 2]$,
we see $|f(x)| \le 1\ \forall\ x \in (0, 2]$

Also $|f(0)| = 0$

$\Rightarrow |f(x)| \le 1\ \forall\ x \in [0, 2]$

43.

$f(0) = 2g(0) = 0, f(1) = 6$

Let $F(x) = f(x) = 2g(x)$

$\Rightarrow F(x)$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$.

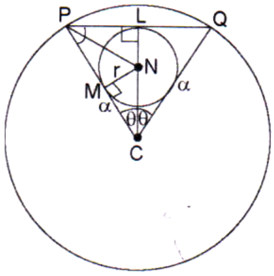
$\therefore$ By L.M.V.T $\exists\ c \in (0, 1)$

Such that $F'(c) = \dfrac{F(1) - F(0)}{1 - 0}$ i.e., $f'(c) - 2g'(c) = [f(1) - 2g(1)] - [f(0) - 2g(0)]$

$\Rightarrow 0 = 6 - 2g(1) - 2 + 0 \Rightarrow g(1) = 2$

44.

In ΔPCN, $PM + MC = \alpha$



$\Rightarrow r\cot\left(\frac{\pi}{4}-\frac{\theta}{2}\right) + r\cot\theta = \alpha$

$\Rightarrow r\left[\cot\left(\frac{\pi}{4}-\frac{\theta}{2}\right)+\cot\theta\right]=\alpha$

$\Rightarrow r\left[\frac{\cot\frac{\theta}{2}+1}{\cot\frac{\theta}{2}-1}+\frac{\cot\theta}{1}\right]=\alpha$

$\Rightarrow r\left[\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}+\frac{\cot\theta}{1}\right]=\alpha$

$\Rightarrow r\left[\frac{1+\sin\theta}{\cos\theta}+\cot\theta\right]=\alpha$

$\Rightarrow r[\sec\theta + \tan\theta + \cot\theta] = \alpha$

$\Rightarrow r = \frac{\alpha}{\sec\theta+\tan\theta+\cot\theta}$

$\Rightarrow r = \frac{\alpha}{\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)}$

$\Rightarrow r = \frac{\alpha\sin\theta\cos\theta}{(\sin\theta+1)}$

$\Rightarrow r = \frac{\alpha}{2}.\frac{\sin 2\theta}{(1+\sin\theta)}$

$\Rightarrow \frac{dr}{d\theta}=\frac{\alpha}{2}.\frac{(1+\sin\theta)(2\cos 2\theta)-\sin 2\theta\cos\theta}{(1+\sin\theta)^2}$

$\therefore \frac{dr}{d\theta} = 0$

$\Rightarrow (1 + \sin\theta)(2 - 4\sin^2\theta) - 2\sin\theta\cos^2\theta$

$\Rightarrow (1 + \sin\theta)[2 - 4\sin^2\theta - 2\sin\theta(1 - \sin\theta)]$

$\Rightarrow (1 + \sin\theta)(-2\sin^2\theta - 2\sin\theta + 2) = 0$

$\Rightarrow \sin\theta = -1$ or $\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-1\pm\sqrt{1+4}}{2}$

$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

45.

$\Delta = \frac{1}{2}$ ab sin θ

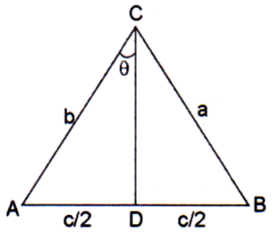
⇒ $\frac{d\Delta}{d\theta} = \frac{1}{2}ab\cos\theta$

∴ For maximum area, $\frac{d\Delta}{d\theta} = 0$

⇒ cos θ = 0

⇒ θ = π/2

⇒ Δ ABC will be a right ∠d Δ, with ∠C = π/2



⇒ $a^2 + b^2 = c^2$ …..(1)

Also, length median AD = $\frac{\sqrt{2a^2+2b^2-c^2}}{2} = \frac{\sqrt{c^2}}{2} = \frac{c}{2}$

⇒ AD = $\frac{\sqrt{a^2+b^2}}{2}$